Complex numbers

* $x^2 = 1 \rightarrow x = \pm\sqrt{1} \rightarrow x = \pm 1$

* $x^2 = -1 \rightarrow x = \pm\sqrt{-1} \rightarrow$ No real Solution.

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = i^2 \times i = -1 \times i = -i$

$i^4 = i^2 \times i^2 = -1 \times -1 = 1$

$i = \sqrt{-1}$ ✓

$i^2 = -1$ ✓

$i^3 = -i$ ✓

$i^4 = 1$ ✓

$i^5 = i^4 \times i = 1 \times i = i$ ✓

$i^6 = i^4 \times i^2 = 1 \times -1 = -1$ ✓

$i^7 = i^4 \times i^3 = 1 \times -i = -i$ ✓

$i^8 = i^4 \times i^4 = 1 \times 1 = 1$ ✓

* $i^{98} = i^2 = -1 \rightarrow 98 \div 4 =$

$$\frac{98}{4} = \frac{80}{4} + \frac{18}{4}$$

$$20 + \frac{16}{4} + \frac{2}{4}$$

$$20 + 4 + \frac{2}{4}$$

$$24\ \frac{2}{4} \leftarrow$$

* $i^{23} = i^3 = -i$

③ $23 \div 4 = \alpha$

$22 \div 4 = \alpha$

$21 \div 4 = \alpha$

$20 \div 4 =$ ✓

Note:

$i^n \rightarrow n \div 4 \rightarrow$ reminder

- 1 → i
- 2 → -1
- 3 → $-i$
- 0 → 1

* $i^{15} = -i$

(3) $\begin{cases} 15 \div 4 = \\ 14 \div 4 = \\ 13 \div 4 = \\ 12 \div 4 = \checkmark \end{cases}$

$\frac{15}{4} = \frac{12}{4} + \frac{3}{4}$

$3 \quad \frac{3}{4}$

* $i^{24} = \quad \frac{24}{4} = \rightarrow (1)$ No reminder.

* $i^{49} \quad 49 \div 4 = \frac{48}{4} + \frac{1}{4} = 12\frac{1}{4} = i$

* $i^{47} \quad 47 \div 4 \Rightarrow \frac{44}{4} + \frac{3}{4} \rightarrow -i$

→ $i^{2n+4} = i^{2n} \times i^4 = (i^2)^n \times 1 = (-1)^n$

* standard form:

$Z = a + bi$ ←

real part ↙ ↘ imaginary number.

$Z = 5 + 3i$ → $a = 5$, $b = 3$

$Z = 7i - 2$ → $a = -2$, $b = 7$

$Z = 5i$ → $a = 0$, $b = 5$

→ Sum & Difference:

$(3 + 5i) + (7 - 2i)$

$10 + 3i$ ←

$(7 - 2i) - (3 + 4i)$

$4 + 2i$ ←

Multiplication:

$5i(2 + 3i) = 10i + 15i^2$ ←

$= 10i + 15(-1)$

$10i - 15$

$3i(7 - 4i) = 21i - 12i^2$

$= 21i + 12$

$(3+2i)(5-7i) = 15 - 21i + 10i - 14i^2$ $+14$

$= 29 - 11i$

$(5-4i)(2+6i) = 10 + 30i - 8i - 24i^2$ $+24$

$= 34 + 22i$

$(5-3i)(5+3i) = 25 + 15i - 15i - 9i^2$ $+9$

$25 + 9$ ← (34)

$(7+4i)(7-4i) = 49 - 28i + 28i - 16i^2$ $\hookrightarrow 16$

$49 + 16 = 65$

Note $(a+bi)(a-bi) = a^2 + b^2$

$(4-3i)(4+3i) = (4)^2 + (3)^2 = 16 + 9 = 25$

$a+bi \longrightarrow a-bi$ (conjugate)

$3+4i \longrightarrow 3-4i$

$5-7i \longrightarrow 5+7i$

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* $\frac{5}{3+2i} \rightarrow \frac{5}{3+2i} \times \frac{3-2i}{3-2i} = \frac{15-10i}{9+4}$

$= \frac{15}{13} - \frac{10i}{13}$

* $\frac{7}{2-3i} \rightarrow \frac{7}{2-3i} \times \frac{2+3i}{2+3i} = \frac{14+21i}{4+9}$

$$\frac{14}{13} + \frac{21}{13}i$$

Ex: $\frac{5+4i}{3+2i} \rightarrow$

$$\frac{5+4i}{3+2i} \times \frac{3-2i}{3-2i} = \frac{15-10i+12i-8i^2}{9+4}$$

($-8i^2$: $+8$)

$$= \frac{23+2i}{13}$$

$$= \frac{23}{13} + \frac{2}{13}i$$

Ex: $\frac{4+3i}{7+5i} \rightarrow$

$$\frac{4+3i}{7+5i} \times \frac{7-5i}{7-5i} = \frac{28-20i+21i-15i^2}{49+25}$$

($-15i^2$: $+15$)

$$= \frac{43+i}{74}$$

$$= \frac{43}{74} + \frac{1}{74}i$$

Ex: $i^3 + i^4 + i^5 + i^6 =$

$-i + 1 + i - 1 = 0$

$i^{10} + i^{11} + i^{12} + i^{13}$

$= 0$ ✓