Exponents:

m ⟶ Exponent (Power)

* a ⟶ base

→ $3^2 = 3 \times 3 = 9$

→ $5^3 = 5 \times 5 \times 5$

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* $a^m \times a^n = a^{m+n}$
* $a^m \div a^n = a^{m-n}$
* $a^0 = 1 \quad , a \neq 0$
* $(a^m)^n = a^{mn}$

Ex: $x^2 \cdot x^3 = x^5$

* If $x^2y^3 = 7$, $xy^2 = 5$ find x^3y^5 ?

$= 35$

$x^2y^3 \cdot xy^2 = x^3y^5$

$7 \times 5 = 35$

Ex: If $3^x = 10$, Then find 3^{x-3} ?

$3^{x-3} = \frac{3^x}{3^3} = \frac{10}{27}$

Ex: $(x^2)^3 = x^6$, $(x^4)^5 = x^{20}$

If $x^2 = 3$ find x^6 ?

* $(x^2)^3 = (3)^3$

$x^6 = 27$

* $x^6 = x^2 \cdot x^2 \cdot x^2$

$= 3 \times 3 \times 3 = 27$

* $(ab)^m = a^m \cdot b^m$

$(3x)^2 = 9x^2$

$(5x^2)^3 = 125x^6$

Note: $\rightarrow x^{2^3} = x^8$, $(x^2)^3 = x^6$

* $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$

Note: $(a \pm b)^m \neq a^m \pm b^m$ ✗

Ex: $\left(\frac{2xy^2}{3mk^4}\right)^2 = \frac{4x^2y^4}{9m^2k^8}$

* Negative Exponents:

$a^{-m} = \frac{1}{a^m}$, $x^{-2} = \frac{1}{x^2}$

$\frac{1}{a^{-m}} = a^m$ $\quad \frac{1}{x^{-5}} = x^5$

$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$ $\quad \left(\frac{x}{y}\right)^{-4} = \left(\frac{y}{x}\right)^4$

Ex: $\frac{5x^{-2}y^3}{4y^{-2}x^4} = \frac{5 \cdot y^2 \cdot y^3}{4 \cdot x^2 \cdot x^4} = \frac{5y^5}{4x^6}$

$= \frac{5}{4} x^{-2-4} y^{3-(-2)}$

$= \frac{5}{4} x^{-6} \cdot y^5 = \frac{5y^5}{4x^6}$

Note: $\rightarrow a^{\frac{m}{n}} = b \rightarrow a = b^{\frac{n}{m}}$

$x^{2/3} = 5 \rightarrow x = 5^{3/2}$

$y^{1/2} = 7 \rightarrow y = 7^2 = 49$

Ex: If $a^{-1/2} = 3$ Then find a?

$$a = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

Ex: If $x^{-3} = 5^{1/3}$ Find x?

$$x = (5^{1/3})^{-1/3} = (5)^{-1/9} = \frac{1}{(5)^{1/9}}$$

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* Radicals:

* $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ $\rightarrow \sqrt{2} \times \sqrt{3} = \sqrt{6}$

$\sqrt{5} \times \sqrt{x} = \sqrt{5x}$

* $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{a/b}$, $b \neq 0$ $\rightarrow \frac{\sqrt{6}}{\sqrt{2}} = \sqrt{\frac{6}{2}} = \sqrt{3}$

* $\sqrt{a^2 b} = a\sqrt{b}$ $\rightarrow \sqrt{3x^2} = \sqrt{3} \times \sqrt{x^2} = x\sqrt{3}$

$\rightarrow \sqrt{27} = \sqrt{9 \times 3} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$

$\rightarrow \sqrt{54} = \sqrt{9 \times 6} = \sqrt{9} \times \sqrt{6} = 3\sqrt{6}$

* $\sqrt{a} \times \sqrt{a} = a.$ $\rightarrow \sqrt{5} \cdot \sqrt{5} = 5$

$\sqrt{x} \cdot \sqrt{x} = \sqrt{x^2} = (\sqrt{x})^2 = x$

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* Square numbers:

$1 = 1^2$	$121 = 11^2$
$4 = 2^2$	$144 = 12^2$
$9 = 3^2$	$169 = 13^2$
$16 = 4^2$	$196 = 14^2$
$25 = 5^2$	$225 = 15^2$
$36 = 6^2$	$400 = 20^2$
$49 = 7^2$	$625 = 25^2$
$64 = 8^2$	
$81 = 9^2$	
$100 = 10^2$	

Ex:

If $\sqrt{x-2} = 5$, Then Find x?

$(\sqrt{x-2})^2 = (5)^2$

$x - 2 = 25$

$x = 27$

If $(\sqrt{2x-1})^2 = (7)^2$ Find x?

$2x - 1 = 49$

$2x = 50 \quad \div 2$

$x = 25$

If $\sqrt{4x+3} + 7 = 10$ Find x?

$(\sqrt{4x+3})^2 = (3)^2$

$4x + 3 = 9$

$4x = 6$

$x = 6/4 = 3/2$

If $\sqrt{x} + \sqrt{9} = \sqrt{64}$ Find x?

$\sqrt{x} + 3 = 8$

$(\sqrt{x})^2 = (5)^2$

$x = 25$

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* $\sqrt{x-2} = x - 4$ Find x?

$\sqrt{4} = 2$

$\sqrt{0} = 0$

$\sqrt{-9} = \alpha$

A) 3 α

B) 2 α

C) 6 ✓

D) $\{3, 6\}$ α

A.M ←

$x - 4 \geq 0$

$x \geq 4$

Radicals & Exponents.

$\rightarrow \sqrt[n]{x^m} = x^{m/n} \leftarrow$

$\rightarrow \sqrt[3]{x^5} = x^{5/3}$

$\rightarrow \sqrt[4]{x^3} = x^{3/4}$

$\rightarrow y^{5/4} = \sqrt[4]{y^5}$

$\frac{3}{2} \cdot \frac{1}{2} = \frac{3}{4}$

$\rightarrow x^{3/4} \cdot (x^{1/2})^{3/2}$, which of the following is true?

A) $\sqrt[3]{x^2}$

B) $\sqrt{x^3}$ ✓

C) $\sqrt[4]{x^5}$

D) $\sqrt[5]{x^4}$

$x^{3/4} \cdot x^{3/4} = x^{\frac{6}{4}} = x^{3/2} = x\sqrt{x^3}$

Note: $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

$\frac{3}{4} + \frac{3}{4} = \frac{6}{4} = 3/2$

$\frac{a}{b} \times \frac{c}{d} = \frac{ad+bc}{bd}$

$\frac{3}{2} \times \frac{5}{7} = \frac{21+10}{14} = \frac{31}{14}$