Trignometry.

Triangles.

Sides

1) Scalene.
2) Isoceles
3) Equilateral

Angles.

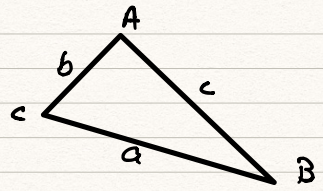
1) Acute
2) Right
3) obtuse.

* Sides:

* Scalene:

$a \neq b \neq c$

$m(\angle A) \neq m(\angle B) \neq m(\angle C)$



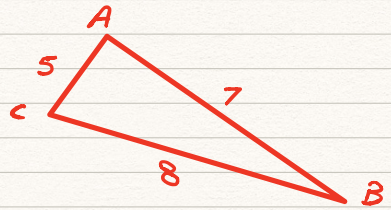
→ Area = $\sqrt{S(S-a)(S-b)(S-c)}$

S = ½ Perimeter.

a, b, c → sides.

Ex:

Find area?



1) $P = 5 + 7 + 8 = 20$

$S = \frac{1}{2}(20) = 10$

Area = $\sqrt{10(10-5)(10-7)(10-8}$ → Calc.

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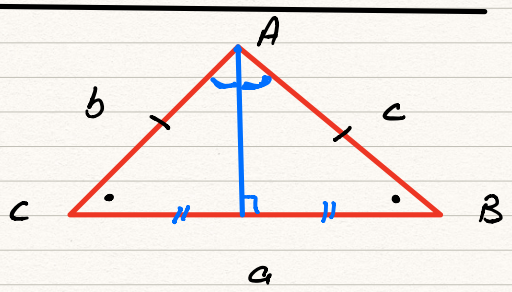
* Isoceles:

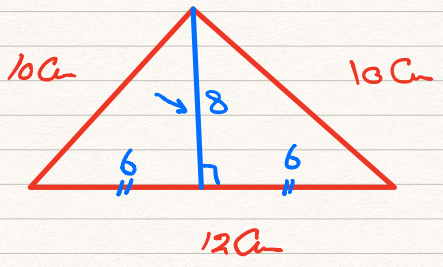
$b = c$

$\hat{B} \cong \hat{C}$



Area = ½ b.h

Ex: Find area?



Area = $\frac{1}{2} b.h$

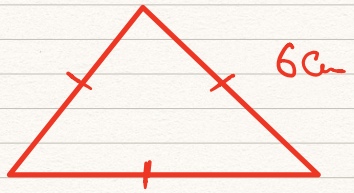
$= \frac{1}{2}(12)(8) = 48\,Cm^2$

Equilateral.

Area = $\frac{1}{2} b.h$

$= \frac{\sqrt{3}}{4} (side)^2$

Ex: Find area?



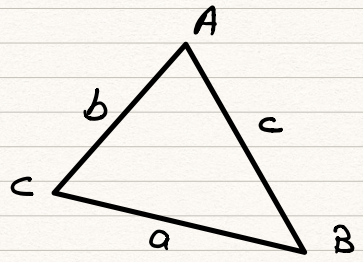
$A = \frac{\sqrt{3}}{4}(6)^2 = 9\sqrt{3}\ Cm^2$

Inequality of triangle:

1) $a + b > c$

$b + c > a$

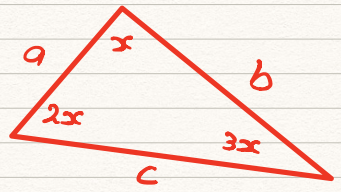
$a + c > b$



2) Greatest angle opposite to greatest side.

3) $a - b < c < a + b$

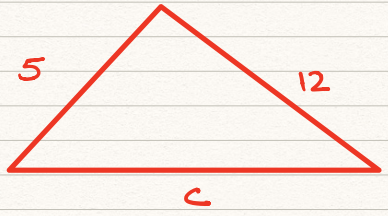
Ex.. Arrange sides ascending?



$x < 2x < 3x$

$c < b < a$

Ex.. Find Possible value for c?



$12 - 5 < c < 12 + 5$

$7 < c < 17$ ←

Ex.. which of the following represent sides of triangle?

a) 3, 1, 4 → $3+1=4=4$ ✗

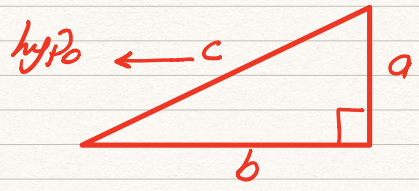
b) 5, 7, 20 → $5+7=12<20$ ✗

c) 2, 3, 5 → $2+3=5=5$ ✗

d) 1, 7, 6 → $1+6=7=7$ ✗

e) 5, 6, 9 → $5+6=11>9$ ✓

→ Pythagorean Theorem:



$c = \sqrt{a^2+b^2}$

$a = \sqrt{c^2-b^2}$

$b = \sqrt{c^2-a^2}$

Triples:

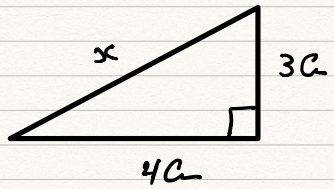
3, 4, 5

5, 12, 13

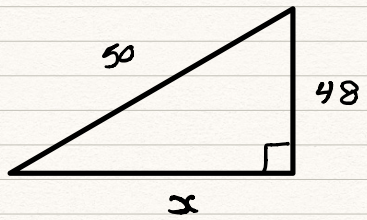
7, 24, 25

8, 15, 17

Ex:



$x = 5\ Cm$



$x = \sqrt{50^2 - 48^2}$

$x = 14\ Cm.$

Pythagorean inequality:

→ a, b, c → c → greatest

$c^2 < a^2 + b^2$ acute

$c^2 = a^2 + b^2$ Right

$c^2 > a^2 + b^2$ obtuse.

→ Ex: Determine type of triangle with sides lengths of 4, 5, 7 ?

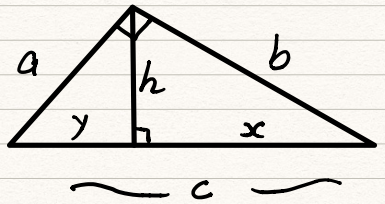
* Scalene.

$7^2 = 49$ $4^2 + 5^2 = 16 + 25 = 41$

$49 > 41$ → obtuse.

Scalene, obtuse.

* Euclidean:



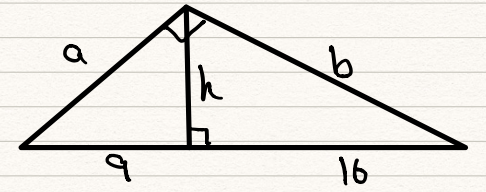
$a^2 = y \cdot c$

$b^2 = x \cdot c$

$h^2 = x \cdot y$

$$h = \frac{ab}{c}$$

Ex.: Find:

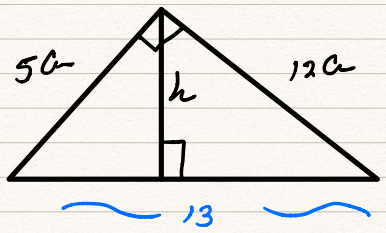


$a = \sqrt{9 \times 25} = 15$

$b = \sqrt{16 \times 25} = 20$

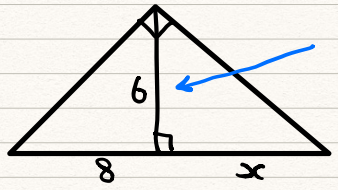
$h = \sqrt{9 \times 16} = 12$

Ex.: Find h?



$$h = \frac{5 \times 12}{13}$$

Find x?

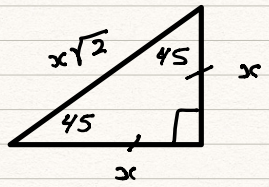


$6^2 = 8x$

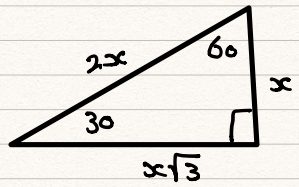
$\frac{9}{2} = \frac{36}{8} = x$

* Special triangle:

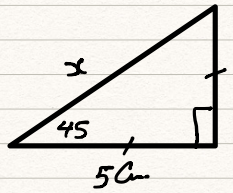
* 45-45-90



* 30-60-90

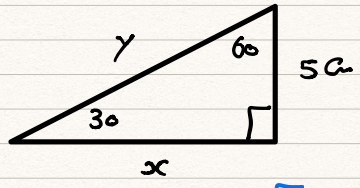


Exc.



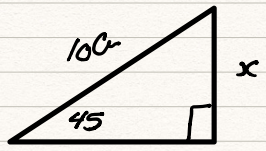
Find x ?

$x = 5\sqrt{2}$

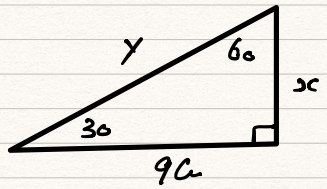


$x = 5\sqrt{3}$

$y = 10$



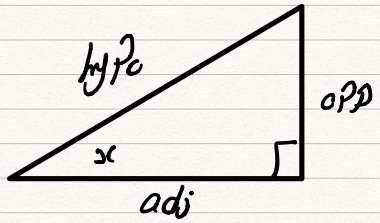
$x = \frac{10}{\sqrt{2}} = 5\sqrt{2}$



$x = 9\sqrt{3} = 3\sqrt{3}$

$y = 6\sqrt{3}$

Trigonometric Ratios:



$\sin(x) = opp/hypo$

$\cos(x) = adj/hypo$

$\tan(x) = opp/adj$

↳ $\sin/\cos$

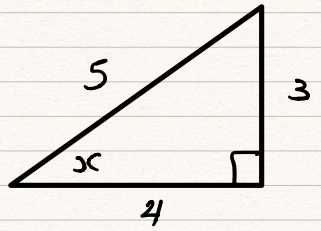
Reciprocals:

$\csc(x) = 1/\sin$

$\sec(x) = 1/\cos$

$\cot(x) = 1/\tan$

Ex:



$\sin(x) = 3/5$

$\cos(x) = 4/5$

$\tan(x) = 3/4$

$\sec(x) = 5/4$

$\csc(x) = 5/3$

$\cot(x) = 4/3$

Note:

$x + y = 90^\circ$

$\sin(x) = \cos(y)$

$\cos(x) = \sin(y)$

$\tan(x) = 1/\tan(y)$

Ex: If $\sin x = \cos 20^\circ$ Find x?

$x + 20 = 90$

$x = 70$

Ex: If $\sin(90 - x) = 3/5$

Then Find $\cos(x) = 3/5$

$90 - x + x = 90$

Ex: If $\sin(2k + 30) = \cos(3k + 10)$, Find k?

$2k + 30 + 3k + 10 = 90$

$5k = 50$

$k = 10$

→ Angles:

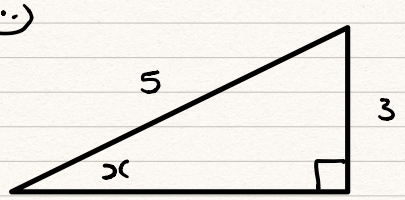
Inverse:

$\sin^{-1}$ (opp / hypo)

$\cos^{-1}$ (adj / hypo) → θ

$\tan^{-1}$ (opp / adj)

Ex.:

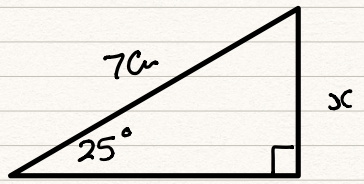


Find x?

$\sin^{-1}(3/5) =$ ✓✓

$\sin^{-1}$ → shift sin.

Ex:

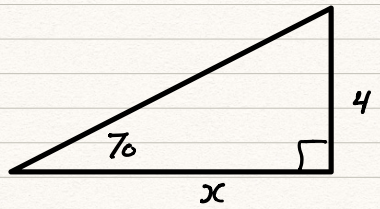


Find x?

$\sin(25) = \frac{x}{7}$

$x = 7 \sin(25)$

Ex:



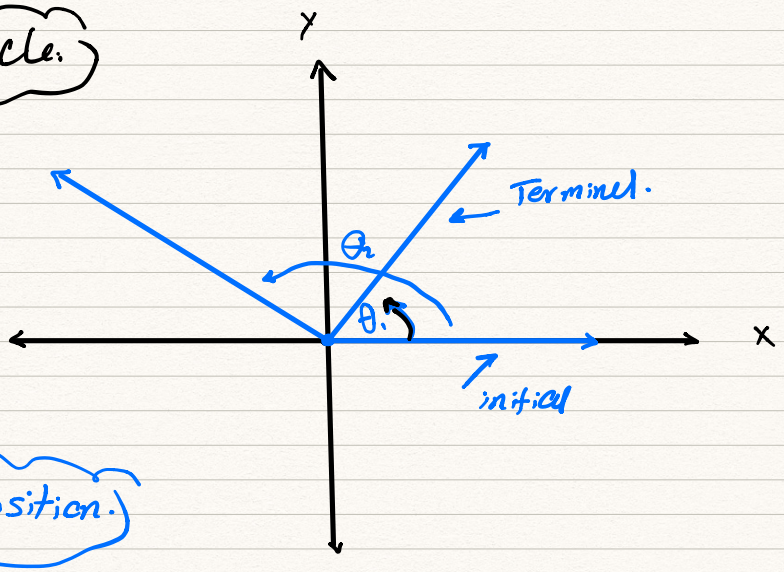
Find x?

$\tan(70) = \frac{4}{x}$

$x = \frac{4}{\tan(70)}$

Unite Circle.

$r = 1$



* Standard Position.

(Angle)

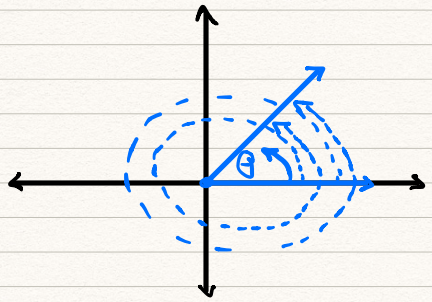
* Vertex → (origin)
* Initial Side → +ve x-axis.

* Coterminal Angle:

$\theta \rightarrow 360 + \theta$

$\rightarrow 2(360) + \theta$

$\rightarrow 3(360) + \theta$



Ex:

$30^\circ \longrightarrow$ Coterminals:

$30 - 360 = -330$

$30 + 360 = 390$

$30 + 2(360) = 750$

Ex: If $m(\angle A) = 1500^\circ$, $\hat{B}$ Coterminal for $\hat{A}$ Then find $m(\angle B)$?

1) $1500 - 360 = 1140$

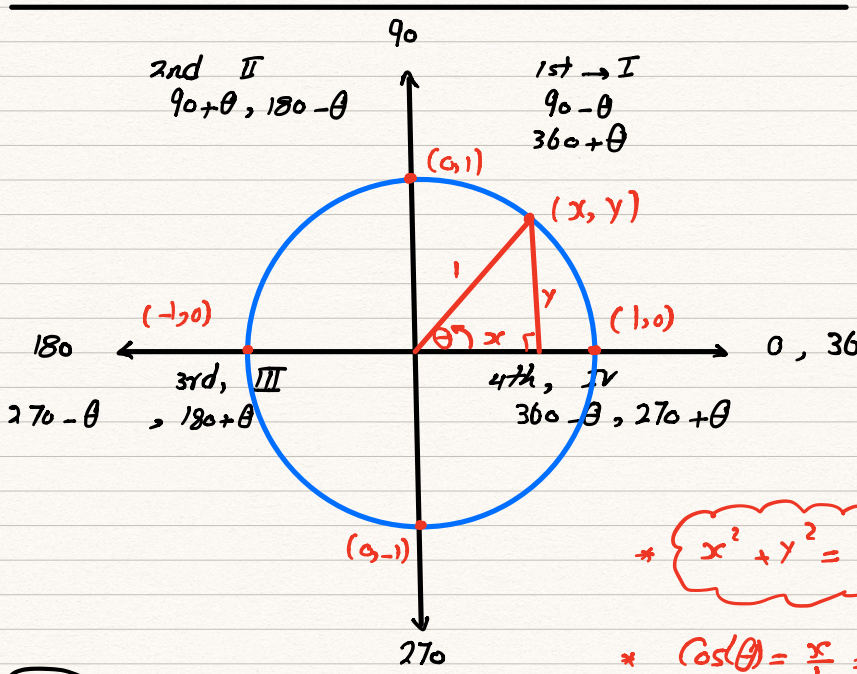
$1140 - 360 = 780$

$780 - 360 = 420$

$420 - 360 = 60$ ✓

2) $\frac{1500}{360} = 4.\cancel{16}$

$1500 - 4(360) = 60$



$r = 1$ unit.

* $x^2 + y^2 = 1$

* $\cos(\theta) = \frac{x}{1} = x$

* $\sin(\theta) = \frac{y}{1} = y$

→ Identities:

$\cos^2(\theta) + \sin^2(\theta) = 1$ ÷ $\cos^2\theta$ ÷ $\sin^2\theta$

$1 + \tan^2(\theta) = \sec^2(\theta)$

$\cot^2\theta + 1 = \csc^2(\theta)$

$\cos^2\theta = 1 - \sin^2\theta$

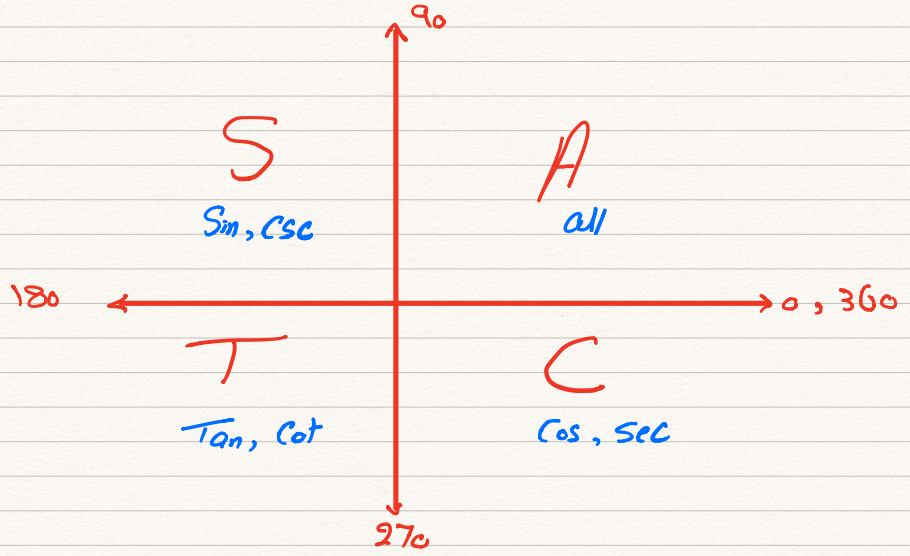
$\sin^2\theta = 1 - \cos^2\theta$

Simplify:

$$\frac{\cos^2\theta}{1-\sin\theta} \overset{??}{=} \frac{1-\sin^2\theta}{1-\sin\theta}$$

$$= \frac{(1-\sin\theta)(1+\sin\theta)}{1-\sin\theta}$$

$$= 1+\sin\theta$$



Note:

90, 270 → Change.

Sin ⇆ Cos

Tan ⇆ Cot

Sec ⇆ Csc

$\sin(90-\theta) = \cos(\theta)$

$\cos(90+\theta) = -\sin\theta$

$\tan(180-\theta) = -\tan(\theta)$

$\cot(180+\theta) = \cot\theta$

$\csc(270+\theta) = -\sec\theta$

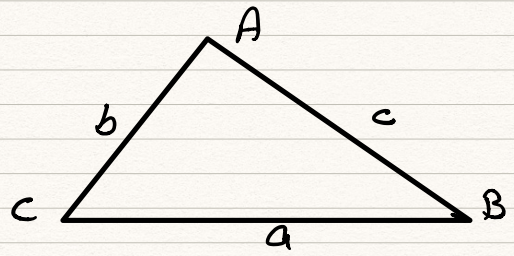
1) Angle ✓
2) Quadrant
3) Sign ✓
4) Change.

Ex:) $\cos(360+\theta) = \cos\theta$

$\sin(\theta-90) = \sin(\theta-90+360)$

$\sin(270+\theta) = -\cos\theta$

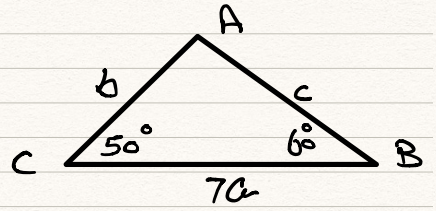
Law of Sine:



* 2 angles, side.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$$

Ex:) Find b, c?



* $m(\angle A) = 180-(50+60)$

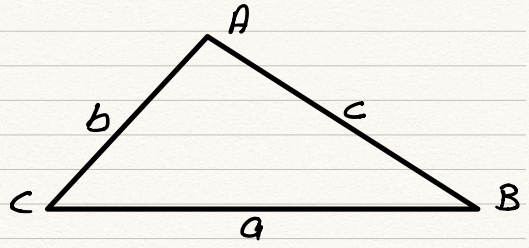
$= 70^\circ$

$$\frac{7}{\sin 70} = \frac{b}{\sin 60} = \frac{c}{\sin 50}$$

$$b = \frac{7\sin 60}{\sin 70}$$

$$c = \frac{7\sin 50}{\sin 70}$$

Law of Cosine:



* 2 Sides, included angle.
* 3 Sides.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

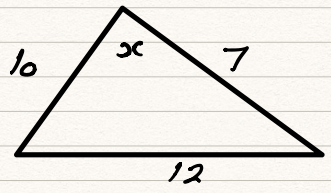
$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

Ex: Find x ?



$$\cos(x) = \frac{10^2 + 7^2 - 12^2}{2 \times 10 \times 7}$$